

# Problem A

## Fading Wind

Time Limit: 1 Second(s)

You're competing in an outdoor paper airplane flying contest, and you want to predict how far your paper airplane will fly. Your design has a fixed factor  $k$ , such that if the airplane's velocity is at least  $k$ , it will rise. If its velocity is less than  $k$  it will descend.

Here is how your paper airplane will fly:

- You start by throwing your paper airplane with a horizontal velocity of  $v$  at a height of  $h$ . There is an external wind blowing with a strength of  $s$ .
- While  $h > 0$ , repeat the following sequence:
  - Increase  $v$  by  $s$ . Then, decrease  $v$  by  $\max(1, \lfloor \frac{v}{10} \rfloor)$ . Note that  $\lfloor \frac{v}{10} \rfloor$  is the value of  $\frac{v}{10}$ , rounded down to the nearest integer if it is not an integer.
  - If  $v \geq k$ , increase  $h$  by one.
  - If  $0 < v < k$ , decrease  $h$  by one. If  $h$  is zero after the decrease, set  $v$  to zero.
  - If  $v \leq 0$ , set  $h$  to zero and  $v$  to zero.
  - Your airplane now travels horizontally by  $v$  units.
  - If  $s > 0$ , decrease it by 1.

Compute how far the paper airplane travels horizontally.

## Input

The single line of input contains four integers  $h, k, v$ , and  $s$  ( $1 \leq h, k, v, s \leq 10^3$ ), where  $h$  is your starting height,  $k$  is your fixed factor,  $v$  is your starting velocity, and  $s$  is the strength of the wind.

## Output

Output a single integer, which is the distance your airplane travels horizontally. It can be shown that this distance is always an integer.

### Sample Input 1

1 1 1 1

### Sample Output 1

1

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**Sample Input 2**

2 2 2 2

**Sample Output 2**

9

**Sample Input 3**

1 2 3 4

**Sample Output 3**

68

**Sample Input 4**

314 159 265 358

**Sample Output 4**

581062

# Problem B

## Creative Accounting

Time Limit: 5 Second(s)

When accounting for the profit of a business, we can divide consecutive days into fixed-sized segments and calculate each segment's profit as the sum of all its daily profits. For example, we could choose seven-day segments to do our accounting in terms of weekly profit. We also have the flexibility of choosing a segment's starting day. For example, for weekly profit we can start a week on a Sunday, Monday, or even Wednesday. Choosing different segment starting days may sometimes change how the profit looks on the books, making it more (or less) attractive to investors.

As an example, we can divide ten consecutive days of profit (or loss, which we denote as negative profit) into three-day segments as such:

$$3, 2, -7 \mid 5, 4, 1 \mid 3, 0, -3 \mid 5$$

This gives us four segments with profit  $-2, 10, 0, 5$ . For the purpose of this division, partial segments with fewer than the fixed segment size are allowed at the beginning and at the end. We say a segment is profitable if it has a strictly positive profit. In the above example, only two out of the four segments are profitable.

If we try a different starting day, we can obtain:

$$3, 2 \mid -7, 5, 4 \mid 1, 3, 0 \mid -3, 5$$

This gives us four segments with profit  $5, 2, 4, 2$ . All four segments are profitable, which makes our business look much more consistent.

You're given a list of consecutive days of profit, as well as an integer range. If we can choose any segment size within that range and any starting day for our accounting, what is the minimum and maximum number of profitable segments that we can have?

### Input

The first line of input has three space-separated integers  $n$ ,  $\ell$  and  $h$  ( $1 \leq \ell \leq h \leq n \leq 3 \times 10^4$ ,  $h - \ell \leq 1,000$ ), where  $n$  is the number of days in the books,  $\ell$  is the minimum possible choice of segment size, and  $h$  is the maximum possible choice of segment size.

Each of the next  $n$  lines contains a single integer  $p$  ( $-10^4 \leq p \leq 10^4$ ). These are the daily profits, in order.

## Output

Output on a single line two space-separated integers  $min$  and  $max$ , where  $min$  is the minimum number of profitable segments possible, and  $max$  is the maximum number of profitable segments possible. Both  $min$  and  $max$  are taken over all possible choices of segment size between  $\ell$  and  $h$  and all possible choices of starting day.

### Sample Input 1

```
10 3 5
3
2
-7
5
4
1
3
0
-3
5
```

### Sample Output 1

```
2 4
```

# Problem C

## Chocolate Chip Fabrication

Time Limit: 1 Second(s)

You are making a chocolate chip cookie using a machine that has a rectangular pan composed of unit squares. You have determined the shape of your cookie, which occupies some squares in that area. Each square of your cookie must be chocolate chipified.

To make the cookie you will repeatedly perform the following two steps:

1. You place cookie dough in some unit squares.
2. You expose the cookie dough to a shallow chocolate chip solution. Any cookie dough square that does not have all four adjacent squares (up, down, left, right) filled with cookie dough becomes chocolate chipified. Note that any cookie dough in a square on the boundary of the pan always gets chipified.

The following example shows how to make a cookie of the shape shown on the left (s):

(s)	(a1)	(a2)	(b1)	(b2)
-X-X-	-D-D-	-C-C-	-C-C-	-C-C-
XXXXX	-D-D-	-C-C-	DCDCD	CCCCC
XXXXX	-DDD-	-CCC-	DCCCD	CCCCC
-XXX-	--D--	--C--	-DCD-	-CCC-
--X--	-----	-----	--D--	--C--

First you place cookie dough in 8 squares (a1). All squares become chipified after the first solution exposure (a2). You place cookie dough in 8 more squares (b1). The second exposure makes every square chipified and completes the cookie (b2).

Your chocolate chip solution is expensive, so you want to ensure that you perform the exposure as few times as possible. Given a cookie shape, determine the minimum number of chocolate chip solution exposures required to make the cookie.

## Input

The first line of input contains two integers  $n$  and  $m$  ( $1 \leq n, m \leq 1,000$ ), indicating the pan has  $n$  rows and  $m$  columns of unit squares.

Each of the next  $n$  lines contains a string of exactly  $m$  characters, where each character is either “X”, representing a square occupied by your cookie, or “-”, representing an empty square.

The shape of your cookie occupies at least one square. Note that the shape may consist of multiple pieces that are disconnected.

## Output

Output the minimum number of chocolate chip solution exposures required to make your cookie.

### Sample Input 1

```
5 5
-X-X-
XXXXX
XXXXX
-XXX-
--X--
```

### Sample Output 1

```
2
```

### Sample Input 2

```
4 5
--XXX
--X-X
X-XXX
XX---
```

### Sample Output 2

```
1
```

### Sample Input 3

```
5 5
XXXXX
XXXXX
XXXXX
XXXXX
XXXXX
```

### Sample Output 3

```
3
```

# Problem D

## Counting Satellites

Time Limit: 1 Second(s)

Nick likes satellites. He likes them so much that he looks for them everywhere. One day he found a string of letters and counted a lot of instances of the word “SATELLITE” among all subsequences of the string. However the next day he forgot this string. Can you help him construct such a string?

String  $s$  is a subsequence of string  $t$  if and only if it is possible to delete some (possibly zero) characters from  $t$  to get  $s$ . Two subsequences are considered different if some character at a given position in  $t$  is deleted in one subsequence but not the other.

### Input

The single line of input contains a single integer  $k$  ( $1 \leq k \leq 10^{18}$ ), which is the number of instances of the word “SATELLITE” in the string Nick forgot.

### Output

Output a string of at most 5,000 uppercase letters. The string must have exactly  $k$  instances of the word “SATELLITE” among all its subsequences. It can be proven that under the given constraints a solution always exists. Note that the length of the string does *not* have to be minimized.

#### Sample Input 1

1

#### Sample Output 1

SATELLITE

#### Sample Input 2

2

#### Sample Output 2

NICKLIKESSATELLITES

#### Sample Input 3

3

#### Sample Output 3

SSSATELLITE

#### Sample Input 4

19

#### Sample Output 4

SATELLITESATELLITE

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# Problem E

## Exponent Exchange

Time Limit: 1 Second(s)

Alice and Bob are playing a cooperative game. They hold  $b^p$  dollars between them, for given integers  $b$  and  $p$ . Alice initially holds  $x$  dollars, and Bob holds  $b^p - x$  dollars. Alice and Bob want to consolidate their money, so one person holds all the money.

In each transaction, one player can choose to send the other player  $b^y$  dollars, for some integer  $y$  with  $0 \leq y < p$ . But each player wants to initiate as few transactions as possible. They are willing to cooperate such that the player that initiates the most transactions (the busiest player), initiates as few as possible.

Alice and Bob want to know the fewest number of transactions that the busiest player needs to initiate to complete the transfer.

### Input

The first line of input contains two integers  $b$  ( $2 \leq b \leq 100$ ) and  $p$  ( $2 \leq p \leq 1000$ ), where  $b$  is the base, and  $p$  is the number of digits.

The next line contains  $p$  integers  $x_{p-1}, x_{p-2}, \dots, x_0$ , separated by spaces, with  $0 \leq x_i < b$  and  $0 < x_{p-1}$ . These are the base- $b$  digits of the value of  $x$ , with the most significant digit first. Specifically,  $x = \sum_{0 \leq i < p} b^i x_i$ . Note that they are given in order from the highest power to the lowest. For example, in the sample, 4 2 7 8 6 with  $b = 10$  represents the base 10 number 42,786.

### Output

Output a single integer, which is the minimum number of transactions the busiest player must initiate to transfer all the money to either Alice or Bob.

#### Sample Input 1

```
10 5
4 2 7 8 6
```

#### Sample Output 1

```
7
```

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# Problem F

## Family Visits

Time Limit: 2 Second(s)

You are a college student living on your own. However, your doting family still likes to visit you, and they often stop by to check on your room at night before going to dinner. Your family will be worried if they find a mess in your room. Therefore you make an effort to ensure that they never see a mess in your room when visiting at night. You have some free time each afternoon that allows you to clean up, but the amount of free time varies each day due to prior commitments.

Luckily, your schedule is planned out well. You know exactly how big of a mess you will make each morning, how much mess you can clean each afternoon, and on which nights your family will stop by. Since you are lazy, you want to spend as few afternoons as possible cleaning such that your family will always see a room without any mess. You may assume that your room starts completely clean, and any mess that is not cleaned remains until it is cleaned.

### Input

The first line of input contains two integers,  $n$  and  $d$  ( $1 \leq d \leq n \leq 1,000$ ), where  $n$  is the number of days in your schedule and  $d$  is the number of days your family will visit.

Each of the next  $n$  lines contains two integers  $m$  and  $c$  ( $0 \leq m, c \leq 1,000$ ). For each day, in order,  $m$  is the amount of mess you make in the morning, and  $c$  is the amount you can clean in the afternoon.

Each of the next  $d$  lines contains a single integer  $v$  ( $1 \leq v \leq n$ ). These are the days on which your family will visit, and they are listed in strictly increasing order.

### Output

Output the smallest number of afternoons you have to spend cleaning to ensure your family will never see a mess. If it is not possible, output  $-1$ .

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**Sample Input 1**

```
6 2
1 2
2 1
1 4
3 2
3 6
2 3
3
6
```

**Sample Output 1**

```
3
```

**Sample Input 2**

```
10 5
12 10
0 2
7 1
1 8
3 4
3 4
2 3
1 2
10 1
7 5
2
4
5
6
8
```

**Sample Output 2**

```
7
```

# Problem G

## Eroding Pillars

Time Limit: 6 Second(s)

You've landed yourself in a real mess, or more accurately a cave filled with dilapidated and unstable pillars. You are luckily standing on a solid rock in the middle of the cave. You know there is a valuable artifact on one of the pillars in the cave, but you aren't sure which one yet. While you wait for the results of a scan of the cave, you start building a robot to help you retrieve the artifact.

The robot you build will be light, and it will assuredly be able to land and jump from any pillar at least once. To build the robot, you need to determine how powerful it needs to be in terms of its jump distance. If the robot is too weak, then it might not be able to reach a pillar. If the robot is too strong, then it will cause a lot of damage when jumping and landing.

Luckily you have already mapped out where all the pillars are located with respect to your starting point at  $(0, 0)$ . You don't know yet which pillar contains the artifact, and you need to finish building the robot first before the scan finishes.

Given the locations of the pillars, you'd like to determine the smallest jump distance to guarantee that your robot can reach any pillar and return back to the start, without landing on a pillar twice.

### Input

The first line of input contains a single integer  $n$  ( $1 \leq n \leq 1,000$ ), which is the number of pillars.

Each of the next  $n$  lines contains two integers,  $x$  and  $y$  ( $-10^9 \leq x, y \leq 10^9$ ). These are the  $(x, y)$  coordinates of the pillars. All pillar locations will be distinct, and no pillar will be at  $(0, 0)$ .

### Output

Output a single number, which is the minimum jump distance needed to guarantee that a robot could reach any pillar and return back to your starting point, without landing on a pillar twice. Your answer will be accepted if it has an absolute or relative error of at most  $10^{-6}$ .

#### Sample Input 1

```
2
1 1
1 0
```

#### Sample Output 1

```
1.414213562373095
```

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**Sample Input 2**

```
8
1 1
0 1
1 0
2 0
0 2
2 1
1 2
2 2
```

**Sample Output 2**

```
1.0
```

# Problem H

## Triangle Containment

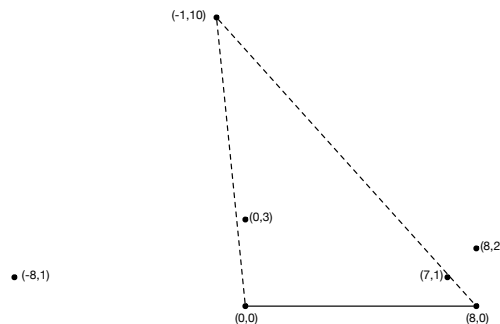
Time Limit: 3 Second(s)

You recently discovered there is treasure buried on your farm land. A **lot** of treasure! You quickly decide to put a fence around the land.

Alas, you have but a single fence post! You will have to drive to town to get more fencing material. But you can't just leave the land as open as it is, so you decide to create a makeshift fence to protect some of the treasure while you are gone. You will place the post in the ground and run some wire in a straight line between two sides of your barn wall and the fence post to section off a triangular area. Also, the ground is very hard: only places that were dug up to bury a treasure are soft enough for you to quickly place the fence post.

To figure out the best option, you first calculate the following. For each of the treasures in your field, if you were to place the fence post at that treasure and complete the fence as described, then what is the total value of all treasures that would be enclosed by the fence? Note that the treasure under the post you place is not considered enclosed by the fence (it might not be safe since someone could dig around the post).

Sample Input 1 is illustrated below. The triangle that includes the point  $(-1, 10)$  encloses exactly two other treasure points which have total value  $4 + 8 = 12$ .



## Input

The first line of input contains two integers  $n$  ( $1 \leq n \leq 10^5$ ) and  $x$  ( $1 \leq x \leq 10^9$ ), where  $n$  is the number of treasure points and  $x$  fixes the two corners of the barn wall at locations  $(0, 0)$  and  $(x, 0)$ .

Each of the next  $n$  lines contains three integers  $x$ ,  $y$ , and  $v$  ( $-10^9 \leq x \leq 10^9$ ,  $1 \leq y \leq 10^9$ , and  $1 \leq v \leq 10^9$ ) giving the location  $(x, y)$  and value  $v$  of one of the treasure points. All of these points are distinct. It is also guaranteed that for each point, the triangle formed with the barn wall will not contain any other treasure point on its boundary.

## Output

Output  $n$  lines, one for each treasure point in the order of the input. For each point output a single integer, which is the total value of all points in the interior of the triangle that point forms with the barn wall. Note that the value of the point itself should be excluded from this sum.

### Sample Input 1

```
5 8
-8 1 1
-1 10 2
0 3 4
7 1 8
8 2 16
```

### Sample Output 1

```
0
12
0
0
8
```

### Sample Input 2

```
6 6
0 1 1
2 3 10
2 5 100
3 1 1000
3 5 10000
4 5 100000
```

### Sample Output 2

```
0
1000
1010
0
1010
1000
```



# Problem I

## Color Tubes

Time Limit: 1 Second(s)

There is a new puzzle generating buzz on social media—Color Tubes. The rules are relatively simple: you are given  $n + 1$  tubes filled with  $3n$  colored balls. Each tube can hold at most 3 balls, and each color appears on exactly 3 balls (so there are  $n$  colors).

Using a series of moves, you are supposed to reach a Color Tubes state—each tube should either hold balls of a single color or it should be empty.

The only move allowed is to take the top ball from one tube and place it into a different tube that has room for it (i.e. holds at most two balls before the move).

You want to write a program to solve this puzzle for you. Initially, you are not interested in an optimal solution, but you want your program to be good enough to solve any puzzle configuration using at most  $20n$  moves.

### Input

The first line of input contains a single integer  $n$  ( $1 \leq n \leq 1,000$ ), which is the number of colors.

Each of the next  $n + 1$  lines contains three integers  $b$ ,  $m$  and  $t$  ( $0 \leq b, m, t \leq n$ ), which are the descriptions of each tube, where  $b$  is the color of the ball on the bottom,  $m$  is the color of the ball in the middle, and  $t$  is the color of the ball on the top.

The tubes are numbered from 1 to  $n + 1$  and are listed in order. The colors are numbered from 1 to  $n$ . The number 0 describes an empty space. It is guaranteed that no empty space will be below a colored ball.

### Output

On the first line output an integer  $m$ , the number of moves that your program will use to solve the puzzle. Remember,  $m$  has to be at most  $20n$ .

On the next  $m$  lines, output two space-separated integers  $u$  and  $v$  that describe a move ( $1 \leq u, v \leq n + 1$ ). In each move, you are taking the uppermost ball out of tube  $u$  and placing it in tube  $v$ , where it will fall until it hits the uppermost ball already in that tube, or the bottom of the tube if the tube is empty.

Your solution will be deemed incorrect if it uses more than  $20n$  moves, or any of the moves are not allowed, or the final configuration is not a Color Tubes state.

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**Sample Input 1**

```
3
2 2 0
1 3 1
3 1 2
3 0 0
```

**Sample Output 1**

```
6
3 1
2 3
2 4
3 2
3 2
3 4
```

**Sample Input 2**

```
1
0 0 0
1 1 1
```

**Sample Output 2**

```
0
```

# Problem J

## Three Dice

Time Limit: 5 Second(s)

Given a list of three-letter words, generate one possible set of three, six-sided dice such that each word can be formed by the top faces of some arrangement of the three dice. You must distribute 18 distinct letters across the 18 total faces of the dice. There may be multiple possible sets of dice that satisfy the requirement; any correct set will be accepted.

### Input

The first line of input contains an integer  $n$  ( $1 \leq n \leq 1,000$ ), which is the number of words.

Each of the next  $n$  lines contains one three-letter word made up only of lowercase letters (a–z). There may be duplicate words in the list, and the words might contain identical letters.

### Output

Output a single line. If there exists a set of dice that can form all of the words, output any such set. Output the set of dice as one line with three space-separated strings, each consisting of six lowercase letters. If no such set of dice can be formed, output a single line with the number 0.

#### Sample Input 1

```
3
lad
fin
sly
```

#### Sample Output 1

```
zounds plight fakery
```

#### Sample Input 2

```
1
dad
```

#### Sample Output 2

```
0
```

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**Sample Input 3**

```
11
aft
cog
far
irk
kit
yes
tau
rag
own
uke
via
```

**Sample Output 3**

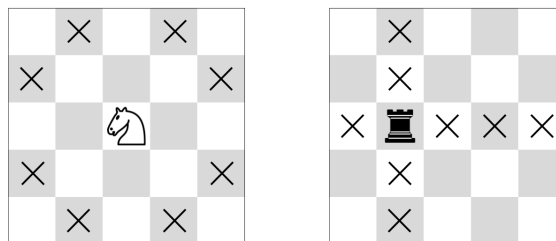
```
vortex whacky fusing
```

# Problem K

## Lone Knight

Time Limit: 7 Second(s)

In the game of chess, a knight moves as shown in the picture below; each move is one square horizontally and two squares vertically or two squares horizontally and one square vertically. A rook can move any number of squares horizontally or vertically, but not both in the same move. If a square can be reached by a rook in one move, that square is said to be attacked by the rook.



Consider an infinite chess board, with squares that can be indexed by integer coordinates. There is a white knight on the board on a square, and it wants to go to another square. However, there are also a number of black rooks on the board. The knight can make as many moves as it needs to get to its target square, but it cannot stop on a square that is attacked by or occupied by a rook. The rooks don't move.

Can the white knight reach its target square? You are to answer that question many times!

## Input

The first line of input contains two integers  $n$  and  $q$  ( $1 \leq n, q \leq 1,000$ ), where  $n$  is the number of black rooks and  $q$  is the number of queries.

Each of the next  $n$  lines contains two integers  $x$  and  $y$ . This indicates that there is a black rook at  $(x, y)$ . No two rooks share the same square.

Each of the next  $q$  lines contains four integers  $x_s, y_s, x_t$  and  $y_t$ . This is a query, where the white knight starts at square  $(x_s, y_s)$  and wants to move to square  $(x_t, y_t)$ .

All square coordinates in the input are no larger than  $10^9$  in absolute value. It is guaranteed that in every query the knight's initial and target squares are not attacked by or occupied by any rook, and the target square is not the same as the initial square.

## Output

For each query, output on a single line 1 if the knight can reach the target square, or 0 otherwise.

### Sample Input 1

```
6 6
10 14
1 0
0 1
4 9
9 13
5 9
2 2 3 4
2 2 2 4
2 2 6 4
2 2 2 10
7 11 6 2
6 2 8 12
```

### Sample Output 1

```
1
0
0
1
0
1
```

### Sample Input 2

```
8 10
0 0
1 1
5 5
8 8
11 11
14 14
18 18
19 19
17 10 13 9
15 15 15 9
7 3 17 4
15 15 12 3
9 17 3 3
4 4 9 4
12 12 2 6
10 15 6 6
15 17 4 16
-1000000000 -999999999 15 7
```

### Sample Output 2

```
1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
```

# Problem L

## Branch Manager

Time Limit: 4 Second(s)

You are managing a transportation network of one-way roads between cities. People travel through the transportation network one by one in order all starting from the same city, and each person waits for the person before them to stop moving before starting. The people follow a simple algorithm until they reach their destination: they will look at all the outgoing roads from the current city, and choose the one that leads to the city with the smallest label. A person will stop when they either reach their destination, or reach a city with no outgoing roads. If at any point someone fails to reach their destination, the rest of the people still waiting in line will leave.

Before each person enters the transportation network, you can permanently close down any subset of roads to guarantee they reach their destination. The roads that you choose to close down will not be available for future people.

There are  $n$  cities, labeled from 1 to  $n$ . There are  $n - 1$  directed roads, and each road will always be from a lower labeled city to a higher labeled one. The network will form a rooted tree with city 1 as the root. There are  $m$  people that want to travel through the network. Each person starts from city 1, and has a specific destination city  $d$  in mind. These people will line up in the given order. What is the maximum number of people you can route correctly to their destination if you close roads optimally?

### Input

The first line of input contains two integers  $n$  and  $m$  ( $2 \leq n, m \leq 2 \times 10^5$ ), where  $n$  is the number of cities and  $m$  is the number of people.

Each of the next  $n - 1$  lines contains two integers  $a$  and  $b$  ( $1 \leq a < b \leq n$ ), denoting a directed road from city  $a$  to  $b$ . These roads will describe a rooted tree with city 1 as the root.

Each of the next  $m$  lines contains a single integer  $d$  ( $2 \leq d \leq n$ ), denoting the destination city of the next person in line.

### Output

Output a single integer, which is the maximum number of people you can route to the correct destination.

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**Sample Input 1**

8 5  
1 2  
4 8  
4 6  
1 4  
2 5  
4 7  
2 3  
5  
2  
6  
4  
8

**Sample Output 1**

5

**Sample Input 2**

4 4  
1 2  
1 3  
1 4  
3  
2  
3  
4

**Sample Output 2**

1



# Problem M

## Alchemy

Time Limit: 1 Second(s)

You just finished day one of your alchemy class! For your alchemy homework, you have been given a string of lowercase letters and wish to make it a palindrome. You're only a beginner at alchemy though, so your powers are limited. In a single operation, you may choose exactly two adjacent letters and change each of them into a different lowercase letter. The resulting characters may be the same as or different from one another, so long as they were both changed by the operation.

Formally, if the string before the operation is  $s$  and you chose to change characters  $s_i$  and  $s_{i+1}$  to produce string  $t$ , then  $s_i \neq t_i$  and  $s_{i+1} \neq t_{i+1}$  must be true, but  $t_i = t_{i+1}$  is permitted.

Compute the minimum number of operations needed to make the string a palindrome.

### Input

The single line of input contains a string of  $n$  ( $2 \leq n \leq 100$ ) lowercase letters, the string you are converting into a palindrome.

### Output

Output a single integer, which is the minimum number of operations needed to make the string a palindrome.

#### Sample Input 1

ioi

#### Sample Output 1

0

#### Sample Input 2

noi

#### Sample Output 2

1

#### Sample Input 3

ctsc

#### Sample Output 3

1

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**Sample Input 4**

fool

**Sample Output 4**

2

**Sample Input 5**

vetted

**Sample Output 5**

2